الامتحان مكون من (5) أسئلة مكتوبة فى صفحة واحدة و المطلوب الإجابة على كل الأسئلة0

Time: 3 Hours

(1)(a)Test the series: (i) $\sum\limits_{n=1}^{\infty} \dfrac{2^n}{2n+1}$ (ii) $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^n}{n^4+1}$

(b)Find the interval of convergence of the series: $\sum\limits_{n=1}^{\infty} \dfrac{(x-2)^n}{\sqrt{n+2}}$

(c)Find the envelope of the curves: $(y+\alpha)^2 + (x-\alpha)^2 = 2$

(2)(a)Find the extrema of the function: $f(x,y) = x^2 + y^3 - 4xy + 4y$

(b)Solve the differential equation: $y`` + y = \tan x$

(3)Solve the differential equations:

(a)$(x^3 + \cos y)dy + (x + 3y\,x^2)dx = 0$ (b)$y``+y = e^{2x} + \cos x$

(c)$x^2 y`` - 3xy` - 5y = x^2$

(4)(a)Solve the system of equations: $(D +1)y - z = x$,

$$- 2Dy + (D +1)z = e^{2x}$$

(b)If $\overline{U} = (2xy)i + (xy\,z^2)j - (xz\,y^2)k$. Find $\nabla.\overline{U}$ and $\nabla x\overline{U}$

(5)(a)Show that the function $u(x,y) = y + e^{2x}\cos 2y$ is harmonic and find its conjugate $v(x,y)$ such that the function $w = u + iv$ is analytic

(b)Evaluate the integrals: (i) $\int\limits_{C} \dfrac{z\cos 3z}{z^2 + 36}dz$ (ii) $\int\limits_{C} \dfrac{\sin 2z}{z^3}dz$

where C (in the two integrals) is the ellipse $|z - 4| + |z + 4| = 10$

Good Luck   Dr. Mohamed H. Eid